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CLOSED EQUATION FOR THE PROBABILITY DISTRIBUTION OF VELOCITY AND TEMPERATURE DIFFERENCES BETWEEN TWO POINTS IN AN ISOTROPICALLY TURBULENT STREAM OF AN INCOMPRESSIBLE FLUID

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A closed equation is derived for the characteristic function describing the joint probability distribution of velocity and temperature differences between two points in an isotropically turbulent stream of an incompressible fluid.

Almost all theories of isotropic turbulence are based either on the formalism of moment equations in any form whatever or on the equation for the characteristic functional. The outcomes of these theories, viz., the closed systems of equations each yields for the spectral energy, are quite similar [1]. An analysis of these equations reveals that they are all incompatible with the mechanism of stagewise energy transfer over the spectrum of fluctuations characterized by successively different scales. The expression for the spectral energy within the inertial range, which can be derived from these equations, contains the mean-square energy of the velocity field. At very high values of the Reynolds number the discrepancy between theory and experiment widens without bounds. This deficiency is overcome by using certain procedures which ensure correct solutions for the inertial range of scales. It is quite doubtful, however, whether the equations thus derived remain valid over the entire universal range. Under consideration here are approximations of the Kraichnan "trial field" kind [2].

The lack of decisive progress made in deriving an equation for the simplest two-point characteristic of isotropic turbulence along conventional lines suggests that new approaches to the problem may have to be tried. One of such approaches could be the formalism based largely on the equations for finite-dimensional functions describing the probability distributions of turbulence fields (F-DFPD formalism, for short). Some developments have already been made so far with regard to this formalism: an array of F-DFPD equations has been derived [3-6], several methods of deriving closed systems of F-DFPD equations have been proposed [7-13], and attempts have been made to analyze the equations for a two-point distribution function covering the inertial scale range, whereupon the corresponding approximate equations have been found to be compatible with the Kolmogorov–Obukhov law [9,11,13]. Closed F-DFPD equations have, furthermore, been derived [14-16] on the basis of a semiempirical theory. The feasibility of developing an analytical theory on the basis of F-DFPD equations has not yet been established, although the results of some studies in this direction [9,11,17,18] are promising.

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Is there any a priori evidence which indicates that this approach may be a successful one? With this approach it is, apparently, possible to utilize to the fullest extent experimentally obtained statistical data on turbulent fields. When any probability distribution function is close to the Gaussian function, e.g., then this can be immediately utilized to the fullest extent by inserting the distribution function in a quasi-Gaussian form into the equation. In the approach based on the formalism of moment equations, on the other hand, one can utilize only some properties of a normal distribution. We further note that the structure of F-DFPD equations, namely the linearity of the term which represents convective transfer, constitutes an important advantage of this approach over the moment approach.

Further analysis will be limited to equations for two-point distribution functions only. The problem of closure at the level of an equation for $f^{(2)}$ reduces to formulation of a hypothesis which allows $f^{(3)}$ to be expressed in terms of $f^{(2)}$ and moments of the latter. When this problem has been somehow solved, there still remains the even more difficult problem of solving a usually nonlinear closed equation for $f^{(2)}$. This function is also multidimensional, since even in the isotropic case it depends on seven arguments. No method of solving such equations has yet been developed. If the equation for $f^{(2)}$ is written in integral form and a step is taken from this equation to equations for the moments, with certain assumptions regarding the form of $f^{(2)}$, however, then it is found possible to obtain closed equations for the moments quite different than the conventional equations based on the formalism of moment equations or of the Hopf equation for the characteristic functional. Here we will attempt to realize such a program of action.

All equations for $f^{(2)}$ which have been derived so far are too intricate for our purpose. For this reason, we will derive here a closed equation for the characteristic two-point function φ describing the joint probability distribution of velocity and temperature differences (with a passive admixture present) in an isotropically turbulent stream of an incompressible fluid. This equation will be relatively simple and well suited for realization of the entire subsequent program of deriving closed equations for the structural functions of turbulent velocity and temperature fields. The characteristic function φ is defined by the expression

$$\varphi \equiv \varphi_{\mathbf{r},\,t}(\boldsymbol{\theta},\ \eta) = \langle \exp[i\theta_\alpha \Delta V_\alpha(\mathbf{r},\ t) + i\eta\Delta T(\mathbf{r},\ t)] \rangle. \tag{1}$$

As the dynamic equations for velocity and temperature fields, we use the Navier–Stokes equations and the equation of convective transfer for the passive admixture we write in the form

$$\frac{\partial \mathbf{V}(\mathbf{x},\ t)}{\partial t} + \mathbf{V}(\mathbf{x},\ t)\cdot\nabla_{\mathbf{x}}\mathbf{V}(\mathbf{x},\ t) = \mathbf{f}(\mathbf{x},\ t) + \nu\Delta_{\mathbf{x}}\mathbf{V}(\mathbf{x},\ t) + \frac{1}{4\pi}\int dx'\ \frac{(\mathbf{x}-\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|^3}\ \nabla_{\mathbf{x}'}\nabla_{\mathbf{x}'} : \mathbf{V}(\mathbf{x}',\ t)\,\mathbf{V}(\mathbf{x}',\ t); \tag{2}$$

$$\frac{\partial T(\mathbf{x},\ t)}{\partial t} + \mathbf{V}(\mathbf{x},\ t)\cdot\nabla_{\mathbf{x}}T(\mathbf{x},\ t) = \varphi(\mathbf{x},\ t) + \chi\Delta_{\mathbf{x}}T(\mathbf{x},\ t). \tag{3}$$

The statistical properties of the random fields $\mathbf{f}(\mathbf{x},\ t)$ and $\varphi(\mathbf{x},\ t)$ must be stipulated. We will assume that these fields are Gaussian and δ-correlated in time [19]. An open equation for φ can be obtained by the method shown in [5,20]. In the same manner, and considering the isotropic case only, we obtain for φ the equation

$$\left(\frac{\partial}{\partial t} - i\nabla_{\mathbf{r}}\nabla_{\boldsymbol{\theta}}\right)\varphi_{\mathbf{r},\,t}(\boldsymbol{\theta},\ \eta) = [\hat{L}\bar{\varphi}_{\mathbf{x},\,t}(\boldsymbol{\theta}',\ \eta'\,|\,\boldsymbol{\theta},\ \eta,\ \mathbf{r})]_0\,\varphi_{\mathbf{r},\,t}(\boldsymbol{\theta},\ \eta) \tag{4}$$

with the notations

$$\hat{L} = -\frac{1}{2}\ \Phi_{\alpha\beta}(\mathbf{r})\,\theta_\alpha\theta_\beta - \frac{1}{2}\ \Psi(\mathbf{r})\,\eta^2 + (\Delta_{\mathbf{x}}^{\mathbf{r}} - \Delta_{\mathbf{x}}^{0})\left[\nu\left(\boldsymbol{\theta}\nabla_{\boldsymbol{\theta}'}\right) + \chi\left(\eta\frac{\partial}{\partial\eta'}\right)\right] - $$
$$- \frac{i}{4\pi}\iint_{\Delta} d\mathbf{x}\,d\mathbf{z}\,\theta_\gamma\left(\frac{x_\gamma}{x^3} + \frac{z_\gamma}{z^3}\right)(\nabla_{\boldsymbol{\theta}'}\nabla_{\mathbf{x}})^2; \tag{5}$$

$$\bar{\varphi} \equiv \bar{\varphi}_{\mathbf{x},\,t}(\boldsymbol{\theta}',\ \eta'\,|\,\boldsymbol{\theta},\ \eta,\ \mathbf{r}) = \frac{\varphi_{\mathbf{r},\,\mathbf{x},\,t}(\boldsymbol{\theta},\ \eta;\ \boldsymbol{\theta}',\ \eta')}{\varphi_{\mathbf{r},\,t}(\boldsymbol{\theta},\ \eta)}. \tag{6}$$

Function $\varphi^{(3)} \equiv \varphi_{\mathbf{r},\mathbf{x},t}(\boldsymbol{\theta},\ \eta;\ \boldsymbol{\theta}',\ \eta')$ in equality (6) is the characteristic function describing the joint probability density of velocity and temperature differences between two points $(0,\ \mathbf{r})$ and of absolute velocity and temperature at a third point $(\mathbf{x})$. Representing $\varphi^{(3)}$ as the product $\bar{\varphi}\varphi$ does not solve the closure problem, of course, but is useful on account of all operations of differentiation and integration programmed in the operator L pertaining to function

$\bar{\varphi}$ only. Equation (4) remains open, because function $\bar{\varphi}$ depends on the statistics at three points.

Function $\bar{\varphi}$ can be expanded into a power series with respect to arguments θ, η, θ', η', if from the corresponding expansion of function $\varphi^{(3)}$ are excluded all terms which relate only to moments depending on vector $\mathbf{r}$. This statement, which provides a prescription for writing approximate expressions for $\bar{\varphi}$, can be easily proved by expanding the logarithms of characteristic functions φ and $\varphi^{(3)}$ in expression (6) into Taylor series.

In order to proceed, it is now necessary to make some assumption regarding the form of function $\bar{\varphi}$. As a workable hypothesis, let this function be a Gaussian one. The corresponding assumption regarding $\varphi^{(3)}$ implies that in the expansion of this function into a Gramme–Charles series the effect of all moments relating to point $\mathbf{x}$ is accounted for in the Gaussian approximation only. The departure from a normal distribution is accounted for by terms in the expansion which relate to points $(0, \mathbf{r})$. Such a rough approximation of function $\bar{\varphi}$ cannot be justified entirely. It is permissible only because in expression (4) there appears not this function itself but its moments (variables θ' and η' are assumed to become zero after operation $\hat{L}$ has been performed). This hypothesis leads to a self-consistent equation for φ.

We write the expression for $\bar{\varphi}$ in the explicit form

$$\bar{\varphi} = \exp\left\{-\frac{1}{2} B(0)\,\theta'^2 + B_{\alpha\beta}(\mathbf{x})\,\theta_\alpha\theta'_\beta - B_{\alpha\beta}(\mathbf{z})\,\theta_\alpha\theta'_\beta - \frac{1}{2} G(0)\,\eta'^2 + G(\mathbf{x})\,\eta\eta' - G(\mathbf{z})\,\eta\eta'\right\}. \tag{7}$$

With such a function $\bar{\varphi}$, Eq. (4) will obviously be a closed one with respect to φ, inasmuch as any of the functions on the right-hand side of expression (7) can be expressed through φ.

Since the functional dependence of $\bar{\varphi}$ on variables θ' and η' has now been completely defined, it is possible to program in expression (5) the operations of differentiation with respect to these two variables and then to equate both variables to zero.

In expression (5) for the operator $\hat{L}$ there also appear operators of differentiation with respect to variable $\mathbf{x}$. This differentiation can be partially realized by using the representation of tensors $B_{\alpha\beta}(\mathbf{x})$ in terms of scalar functions, which is valid in the case of an isotropic helical velocity field [21], and also using expressions which relate structural functions of turbulent fields to the correlation functions of these fields. Simple but somewhat unwieldy calculations yield an equation for $\varphi_{\mathbf{r},t}(\theta, \eta)$ which we will write in the conventional in statistical physics form

$$\left(\frac{\partial}{\partial t} + L_0\right)\varphi_{\mathbf{r},\,t}(\theta,\ \eta) = L(\theta,\ \eta,\ \mathbf{r},\ t)\,\varphi_{\mathbf{r},\,t}(\theta,\ \eta) \tag{8}$$

with

$$L_0 = -i\nabla_{\mathbf{r}}\nabla_{\theta}\,. \tag{9}$$

We then thoroughly explicate the scalar expression $L(\theta, \eta, \mathbf{r}, t)$ with the time argument omitted (for the sake of brevity)

$$L(\theta,\ \eta,\ \mathbf{r},\ t) = \Omega(\mathbf{r})\,\eta^2 + \Omega_{\alpha\beta}(\mathbf{r})\,\theta_\alpha\theta_\beta + i\Omega_{\alpha\beta\gamma}(\mathbf{r})\,\theta_\alpha\theta_\beta\theta_\gamma; \tag{10}$$

$$\Omega(r) = 2\bar{N}_d - \frac{1}{2}\Psi(r) + \chi H''(r)\,X(r); \tag{11}$$

$$\Omega_{\alpha\beta}(\mathbf{r}) = \frac{2}{3}\bar{\varepsilon}_d\delta_{\alpha\beta} - \frac{1}{2}\Phi_{\alpha\beta}(\mathbf{r}) + 2\nu D''(r)\,V_{\alpha\beta}(\mathbf{r}); \tag{12}$$

$$\Omega_{\alpha\beta\gamma}(\mathbf{r}) = -\frac{1}{32\pi}\iint\limits_{\Delta} dx dz\left(\frac{x_\gamma}{x^3} + \frac{z_\gamma}{z^3}\right)\times \\ \times\,[D'(x)^2\,\omega^{(1)}_{\alpha\beta} + D'(x)\,D'(z)\,\omega^{(2)}_{\alpha\beta}]; \tag{13}$$

$$\bar{N}_d = \frac{3}{2}\chi\lim_{\mathbf{r}\to 0} H''(r);\ \bar{\varepsilon}_d = \frac{15}{2}\nu\lim_{\mathbf{r}\to 0} D''(r); \tag{14}$$

$$X(r) = -1 + \frac{1}{2}\delta^{-1}(r);\ V_{\alpha\beta}(\mathbf{r}) = \eta_1(r)\,\Delta_{\alpha\beta}(\mathbf{r}) + \eta_2(r)\,\delta_{\alpha\beta}; \tag{15}$$

$$\eta_1(r) = 1 - \frac{1}{4}\gamma(r) + \beta^{-1}(r);\ \eta_2(r) = -\frac{1}{2} + 2\beta^{-1}(r); \tag{16}$$

$$\delta(r) = -r[\ln H'(r)]';\ \gamma(r) = -r[\ln D''(r)]';\ \beta(r) = -r[\ln D'(r)]'. \tag{17}$$

When the correlation functions of the random fields $f(\mathbf{x}, t)$ and $\varphi(\mathbf{x}, t)$ are taken in the form $F(r) = (2/3)\bar{\varepsilon}\exp(-r^2/L^2)$ and $\Phi(r) = 2\bar{N}\exp(-r^2/L^2)$, respectively, then for $r \ll L$ the structural tensor $\Phi_{\alpha\beta}(\mathbf{r})$ and the structural function $\Psi(\mathbf{r})$ can be expressed as

$$\Phi_{\alpha\beta}(\mathbf{r}) = -\frac{2}{3}\bar{\varepsilon}\frac{r^2}{L^2}\varphi_{\alpha\beta}(\mathbf{r});\ \Psi(r) = 2\bar{N}\frac{r^2}{L^2}. \tag{18}$$

Parameters $\bar{\varepsilon}$ and $\bar{N}$ defining the rate of "energy" pumping up the velocity field and the temperature field are, just as parameter L, obviously external parameters of the problem. The restriction $r \ll L$ is justified by its being exactly the constraint which defines one limit of the universal scale range.

The expression for $\omega_{\alpha\beta}^{(l)}$ is

$$\omega_{\alpha\beta}^{(l)} = P_1^{(l)}\varkappa_\alpha\varkappa_\beta + \frac{1}{2}P_2^{(l)}(\varkappa_\alpha\lambda_\beta + \lambda_\alpha\varkappa_\beta) + P_3^{(l)}\delta_{\alpha\beta},\ l = 1, 2, \tag{19}$$

where

$$P_i^{(l)} = \sum_{n=1}^{4} P_{in}^{(l)}\beta_n; \tag{20}$$

$$P_{in}^{(1)} = \begin{vmatrix} 0 & -2 & 0 & 12 \\ 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -6 \end{vmatrix}; \tag{21}$$

$$P_{in}^{(2)} = \begin{vmatrix} \left[-2\omega-(\omega^2+1)\frac{\sigma}{\xi}\right] & \left[4\omega-(\omega^2-3)\frac{\sigma}{\xi}\right] & \left[-2\omega-(\omega^2-3)\frac{\sigma}{\xi}\right] & \left[4\omega-(\omega^2+7)\frac{\sigma}{\xi}\right] \\ (\omega^2+1)\frac{\rho}{\xi} & (\omega^2-3)\frac{\rho}{\xi} & (\omega^2-3)\frac{\rho}{\xi} & (\omega^2+7)\frac{\rho}{\xi} \\ 0 & 2\omega & 0 & -6\omega \end{vmatrix}; \tag{22}$$

$$\beta_n = \begin{vmatrix} \beta(x)\ \beta(z) \\ \beta(x) \\ \beta(z) \\ 1 \end{vmatrix}; \tag{23}$$

$$\omega = (\boldsymbol{\varkappa}\cdot\boldsymbol{\mu}). \tag{24}$$

Expressions (11)-(24) indicate that expression $L(\theta, \eta, \mathbf{r}, t)$ is completely defined by simultaneous structural functions of velocity and temperature fields and external random fields. Since the simultaneous structural function $D(r)$ of the velocity field is determined by the difference of velocities at instant of time t at points a distance r apart, obviously Eq. (8) will be invariant with respect to Galileo transformations.

Using expression (8) for the derivation of equations for the structural functions $D(r)$ and $H(r)$, we arrive at exact (and, of course, open) equations for these functions, i.e., we find that the equations for $D(r)$ and $H(r)$ obtained from the approximate expression (8) are identical to those which can be obtained directly from the array of moment equations. This is another argument in favor of the chosen approximation.

The characteristic function $\varphi_{r,t}(\theta, \eta)$ must satisfy a number of additional conditions. Those conditions can be stated as

$$(\hat{H}_1\varphi)_0 = 1;\ (\hat{H}_i\varphi)_0 = 0,\ i = 2,\ 3,\ \ldots,\ 6; \tag{25}$$

$$\hat{H}_1 = 1;\ \hat{H}_2 = \nabla_{\theta_\alpha};\ \hat{H}_3 = \frac{\partial}{\partial\eta};\ \hat{H}_4 = \nabla_{r_\alpha}\nabla_{\theta_\alpha};$$
$$\hat{H}_5 = \nabla_{r_\beta}\nabla_{\theta_\beta}\nabla_{\theta_\alpha};\ \hat{H}_6 = \nabla_{r_\beta}\nabla_{r_\gamma}\nabla_{\theta_\alpha}\nabla_{\theta_\beta}\nabla_{\theta_\gamma}. \tag{26}$$

Condition 1 is the condition of normalization, conditions 2, 3, 4 signify zero first moments of velocity and temperature differences in an isotropic stream, conditions 5, 6 are consequences of the helicity of the velocity field.

That function φ defined by expression (8) satisfies conditions 1-6 can be proved as follows. In order that these conditions be satisfied at all times, it is obviously necessary that the derivatives of the left-hand sides of these equalities be zero. Then, if these conditions are satisfied at t = 0, i.e., the characteristic function has been correctly chosen at the start, they will be satisfied at any other time. Formally this implies that

$$\frac{d}{dt}\ (\hat{H}_i\varphi)_0 = \hat{H}_i\left(\frac{d\varphi}{dt}\right)_0 = 0. \tag{27}$$

With the aid of expression (8), we obtain

$$(\hat{H}_i L_0\varphi)_0 + [\hat{H}_i L(\theta,\ \eta,\ \mathbf{r},\ t)]_0 = 0. \tag{28}$$

Considering this condition (28) for each case i = 1, 2, ..., 6 reveals a complete self-consistency of all conditions, if only the approximations

$$\langle\, \Delta V_\alpha(\mathbf{r})\,\Delta V_\beta(\mathbf{r})\,\Delta V_\gamma(\mathbf{r})\,\Delta V_\delta(\mathbf{r})\,\rangle = D_{\alpha\beta}(\mathbf{r})\,D_{\gamma\delta}(\mathbf{r}) + D_{\alpha\gamma}(\mathbf{r})\,D_{\beta\delta}(\mathbf{r}) + D_{\alpha\delta}(\mathbf{r})\,D_{\beta\gamma}(\mathbf{r}); \tag{29}$$

$$\langle\, \Delta V_\alpha(\mathbf{r})\,\Delta T(\mathbf{r})\,\rangle = 0 \tag{30}$$

are made in the calculations. Equality (30) is always satisfied in an isotropic stream of an incompressible fluid [21]. Equality (29) is satisfied as a consequence of the chosen approximation for function $\bar{\varphi}$.

After a Fourier transformation with respect to variables θ, η of Eq. (8), we arrive at an equation for the probability density distribution $P_{r,t}(V,\ T)$ of velocity and temperature differences between two points. Function $P_{r,t}(V,\ T)$ must be a real nonnegative one. This property is ensured by Eq. (8), which can be easily proved exactly as in [8]. The uniquely essential point in the proof is L(**V**, T, **r**, t) being a real quantity. This is evident from expression (10), with real coefficients at the second powers of θ, η and imaginary coefficients at the third powers of these variables.

A very important step is the change from differential to integral form of Eq. (8) for φ. In order to make this change, it is necessary to know the Green function for the left-hand side of Eq. (8). This function is the solution to the equation

$$\left(\frac{\partial}{\partial t} - i\nabla_r\nabla_\theta\right) G(\theta,\ \eta,\ \mathbf{r},\ t) = \delta(\theta)\,\delta(\eta)\,\delta(\mathbf{r})\,\delta(t). \tag{31}$$

Solving this equation by the method in [22] and using the initial condition G(t) = 0, if t < 0, we find that

$$G(\theta,\ \eta,\ \mathbf{r},\ t) = \frac{\Theta(t)}{(2\pi)^3 t^3}\ \exp\left[i\ \frac{(\theta\cdot\mathbf{r})}{t}\right]\delta(\eta). \tag{32}$$

With this expression for Green's function, it is now possible to write Eq. (8) in integral form as

$$\varphi_{\mathbf{r},\,t}(\theta,\ \eta) = \frac{1}{(2\pi)^3}\int_0^t \frac{d\tau}{(t-\tau)^3}\int d\mathbf{y}\int d\theta'\,\exp\left[i\ \frac{(\theta-\theta')(\mathbf{r}-\mathbf{y})}{t-\tau}\right] L(\theta,\ \eta,\ \mathbf{y},\ \tau)\,\varphi_{\mathbf{y},\,\tau}(\theta',\ \eta). \tag{33}$$

The term which represents the initial conditions in explicit form and which decays fast has been omitted here.

In conclusion, we note that the change from Eq. (8) to Eq. (33) is a purely formal and also reversible step. When from Eq. (33) are derived equations for the moments of the characteristic function φ, i.e., for the structural functions D(r) and H(r), however, then the change from Eq. (8) to Eq. (33) becomes "fixed" and irreversible. Such a procedure most easily utilizes the linearity of the convection term in Eq. (8) for the characteristic function. The derivation of closed equations for D(r) and H(r) obviously requires that some statistical hypothesis regarding the form of function φ in Eq. (33) be adopted. The derivation of closed equations for the structural functions on the basis of Eq. (33) will be the next task.

NOTATION

$f^{(n)}$, n-point probability distribution function; $\mathbf{V}(\mathbf{r}, t)$, velocity field; $T(\mathbf{r}, t)$, temperature field; $\Delta V(\mathbf{r})$, velocity difference between points $\mathbf{r}$, 0; $\Delta T(\mathbf{r})$, temperature difference between points $\mathbf{r}$, 0; θ, η, arguments of the characteristic function φ; $f(\mathbf{x}, t)$, external random force; $\varphi(\mathbf{x}, t)$, external random source of temperature nonuniformity; $\Phi_{\alpha\beta}(\mathbf{r})$, structural tensor of the $f(\mathbf{x}, t)$ field; $\Psi(\mathbf{r})$, structural function of the $\varphi(\mathbf{x}, t)$ field; $B_{\alpha\beta}(\mathbf{r})$, correlation tensor of the $V(\mathbf{r})$ field; $B(0)$, turbulence energy per unit mass of fluid; $G(\mathbf{r})$, correlation function of the $T(\mathbf{r})$ field; $D(\mathbf{r})$, longitudinal structural function of the $\mathbf{V}(\mathbf{r})$ field; $H(\mathbf{r})$, structural function of the $T(\mathbf{r})$ field; $\nabla_{\mathbf{r}}$, a gradient; $\Delta_{\mathbf{r}}$, Laplacian; $\Delta_{\mathbf{x}}^{\mathbf{r}} = \Delta_{\mathbf{x}} f_{|\mathbf{x}=\mathbf{r}}$; $\Delta_{\alpha\beta}(\mathbf{r}) = \nu_\alpha \nu_\beta - \delta_{\alpha\beta}$; $\varphi_{\alpha\beta}(\mathbf{r}) = \Delta_{\alpha\beta}(\mathbf{r}) - \delta_{\alpha\beta}$; $\nu = \mathbf{r}/r$; $\lambda = \mathbf{y}/y$; $\varkappa = \mathbf{x}/x$; $\mu = \mathbf{z}/z$; $\rho = y/r$; $\sigma = x/r$; $\xi = z/r$; $\omega = (\varkappa \cdot \mu)$; $\iint_\Delta d\mathbf{x}d\mathbf{z} = \iint d\mathbf{x}d\mathbf{z}\,\delta(\mathbf{x}+\mathbf{z}-\mathbf{r})$; vectors $\mathbf{x}$, $\mathbf{z}$, $\mathbf{r}$ form the triangle $\mathbf{x} + \mathbf{z} - \mathbf{r} = 0$; $\mathbf{ab}:\mathbf{cd} = a_\alpha b_\beta c_\alpha d_\beta$, with a summation from 1 to 3 over all repeated indices everywhere; $\Theta(t) = 1$ when $t > 0$; $\Theta(t) = 0$ when $t < 0$; L, external turbulence scale; ε, rate of pumping up the turbulence energy per unit mass; parameter $\bar{N}$ determines the rate of pumping up the temperature nonuniformity; and $[\,]_0 = [\,]\Big|_{\substack{\theta=0 \\ \eta'=0}}$.

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